

**Doctor of Business Economics (VUB)
Doctor of Economics (UGent)**

**This PhD is part of a cooperation agreement between the Vrije Universiteit
Brussel (VUB) and the Universiteit Gent (UGent).**

**Advances in Robust Covariance Estimation and Optimal Payoff Choice for
Financial Decision Making**

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Abstract

In this thesis, I address the challenges of managing model risk in financial decision making by proposing novel statistical methods for covariance matrix estimation. The widespread accessibility of high-frequency asset price data has catalyzed the creation of techniques aimed at retrospectively estimating covariation within a predetermined time span. I explore various covariance estimators, aiming to improve their accuracy and ensure positive-definiteness. I show their usefulness in the context of minimum tracking error portfolios. I also study in detail the optimization problem for a Yaari investor who seeks to maximize their utility under the constraints of variance minimization, extending the conventional mean-variance framework to incorporate distorted expectation-variance optimization. Furthermore, I investigate the optimal payoff strategy for an investor aiming to surpass an externally defined benchmark. Through the analysis, I contribute to understanding of Yaari's dual theory in various contexts and provide insights into optimizing payoffs for such investors in these distinct scenarios.

The first method introduced is the Basket-Adjusted Covariance (BAC), which leverages information from aggregate portfolios of assets, such as Exchange Traded Funds (ETFs). The BAC refines pre-estimators by adding a minimal adjustment matrix to align the covariance-implied stock-ETF covariation with a target value. The simulation study demonstrates substantial accuracy gains, and empirical results show improved tracking error efficiency when constructing portfolios using BAC.

The second contribution is the Posterior Mean Covariance Estimator, a Bayesian approach to enhance covariance matrix estimation. The Bayesian posterior mean estimator incorporates a likelihood function based on pre-estimator approximation error and a non-informative prior distribution enforcing positive-definiteness. This leads to a closed-form, positive definite improvement, outperforming existing methods in terms of mean squared error (MSE) for various covariance estimators and shrinkage instruments.

The third contribution is the derivation of the optimal pay-off for Yaari investors including the extension of the mean-variance optimization framework to distorted expectation-variance optimization. Additionally, I derive optimal payoffs for investors aiming to outperform external benchmarks while staying within a defined neighborhood of these benchmarks.

Overall, proposed methods contribute to the enhancement of financial decision making by improving the accuracy of covariance matrix estimation. These advancements have broad applications in portfolio optimization and achieving optimal payoffs under specific variance constraints.